Lot Sizing to Minimize Cycle Time in a Sequential Operation

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1 Introduction

We discuss two methods of lot sizing: traditional EOQ lot sizing and a method that seeks to minimize cycle time. The setting is for a "sequential" as opposed to a "simultaneous" operation. Simultaneous operations are those that work on several parts as one time (e.g., heat treat). A sequential operation is one in which a processor that can work on parts sequentially but must spend some time in a setup or changeover before moving to a different part. The classic example is a punch press. These machines can punch shapes into sheet metal at a very fast rate but can take significant amounts of time (time for many parts to be processed) to change from one part to another. The decision of how many parts of a certain "family" to process before switching to a different family requiring a setup (or changeover) involves a tradeoff between capacity and cycle time. Figure 1 illustrates this process.

2 Lot Sizing and Cycle Time

The lot size has a large impact on the system cycle time. The larger the batch the greater the capacity since relatively more time is spent in production rather than in setups. With more capacity, utilization will be lower. One of the principles of Factory Physics is that as utilization approaches 100%, cycle time increases dramatically. Also, once utilization goes below 80% or so, queue time is very low so that increasing lot sizes to increase capacity will have less impact at low utilization levels.

On the other hand, increasing lot sizes too much will cause more waiting in queue when a batch arrives and finds the system busy (it simply takes longer for the current batch to finish). Also, increasing batch sizes will obviously increase the time spent for the batch to finish (i.e., wait-in-batch time). Figure 2 presents all of these components.

There are also some subtleties that arise because, unlike in the simultaneous batching case, in sequential batch operations it is possible to release completed entities to the next operation downstream before the entire batch has been completed. Since these issues change the quantitative behavior but not the qualitative behavior of sequential batching operations, we treat the simplest case here.

Consider a CNC milling machine that takes 6 minutes to produce a part after it is setup but takes 10 hours to setup and verify. Suppose we must make 13,000 units per year. Now, of

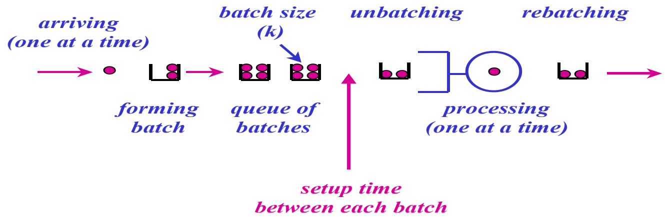

Figure 1: Mechanics of Sequential Batching.

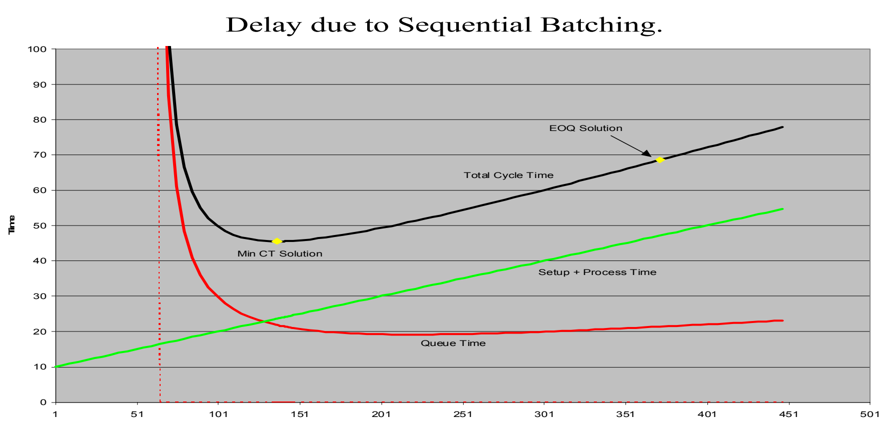

Figure 2: Delay due to sequential batching.

course, before we address what is the best lot size, we should first address the issue of why it takes 10 hours to change over the mill. There are times when changeovers cannot be further reduced (e.g., a change in lubricating fluid, extremely tricky alignments, hot furnaces, detailed cleanings in pharmaceuticals). Suppose this is one such case. What is the best lot size?

3 EOQ Lot Sizing

One way people have done this is to consider setup cost and inventory carrying cost. Suppose the labor cost is \$25 per hour (including fringe, etc.). Then the direct labor cost to do the setup is \$250. Suppose the part cost is \$150, including GSA, engineering support, maintenance, the cost of the mill, etc., but not including the setup cost.[1] Finally suppose the inventory carrying cost ratio is 25% (including cost of capital, cost of storage, spoilage, etc.). Using the "economic order quantity" we arrive at a lot size of

$$Q = \sqrt{\frac{(2)(250)(13,000)}{(150)(0.25)}} = 416$$

This is a "minimum cost" lot size. What are its logistical implications?

4 Minimum Cycle Time Lot Sizing

Now consider the cycle time through the milling area. Suppose t represents the time to do one part (0.1 hour) and s the setup time (10 hours). If Q (416) is the lot size then the time spent at the mill (setup and processing), T, will be

$$T = Qt + s = (416)(0.1) + 10 = 51.6 \text{ hours} \tag{4.1}$$

We must also include queue time. From the VUT equation of Factory Physics,

$$\text{CT} = VUT + T \tag{4.2}$$

The form of the U factor for a single station is

$$U = \frac{u}{1-u} \tag{4.3}$$

where u is the utilization of the station. We obtain the utilization by first computing the arrival rate to the station. With 2 shifts running 8 hours at 86.67% for 250 days per year, we would have an arrival rate, r_a, to the mill of

$$r_a = \frac{13,000}{(8)(0.8667)(2)(250)} = 3.75 \text{ parts/hour}$$

The arrival rate of the batches is simply the arrival rate of the single parts divided by the lot size, r_a/Q. Then the utilization will be the arrival rate of the batches multiplied by the time it

[1] Whether we include the setup cost or not will not change the EOQ.

takes to do a batch, T

$$\begin{aligned} u &= (r_a/Q)T \\ &= (r_a/Q)(Qt+s) \\ &= r_a t + r_a s/Q \\ &= (3.75)(0.1) + (3.75)(10)/416 = 0.4651 \end{aligned} \tag{4.4}$$

a very low utilization so we would expect little queueing. The utilization factor will be

$$U = \frac{u}{1-u} = \frac{0.4651}{1-0.4651} = 0.8695 \tag{4.5}$$

A reasonable V factor in this case is 0.5. Substituting all the above into the VUT equation shows the cycle time to be

$$\text{CT} = VUT + T = (0.5)(0.8895)(51.6) + 51.6 = 74.0 \text{ hours} \tag{4.6}$$

or 10.7 shifts which is over one week! Is this the best we can do? Consider the plot in figure 2 which shows the components of cycle time for the mill. There we see that cycle time can be reduced greatly by reducing the lot size. Apparently, the lot size of 416 was overly conservative (as seen by the resulting utilization). But how do we find a good lot size?

First we should compute the minimum lot size that guarantees less than 100% utilization by considering equation (4.4).

$$\begin{aligned} u &= r_a t + r_a s/Q < 1 \\ &\text{or} \\ r_a s/Q &< 1 - r_a t \\ Q &> \frac{r_a s}{1 - r_a t} \\ &> \frac{(3.75)(10)}{1-(3.75)(0.1)} = 60 \end{aligned} \tag{4.7}$$

So the lot size must be at least 61. However, the figure shows this value to yield a huge cycle time (because of the high effective utilization). Obviously, the utilization must be between the "productive utilization" (i.e., the utilization if there were no setups) and 100% utilization. **Productive utilization**, u_0, is defined as the utilization that would be seen if there were no setups,

$$u_0 = r_a t = (3.75)(0.1) = 0.375 \tag{4.8}$$

A good value for the target effective utilization, u^*, is the square root of the productive utilization,

$$u^* = \sqrt{u_0} = \sqrt{(0.379)} = 0.612 \tag{4.9}$$

Now we need to find the lot size corresponding to this utilization. Again we use equation (4.4).

$$\begin{aligned} u^* &= r_a t + r_a s/Q \\ &= u_0 + r_a s/Q \\ u^* - u_0 &= r_a s/Q \end{aligned} \tag{4.10}$$

So that,

$$\begin{aligned} Q &= \frac{r_a s}{u^* - u_0} \\ &= \frac{(3.75)(10)}{0.612 - 0.375} \approx 158 \end{aligned} \tag{4.11}$$

This yields a cycle time of 46.2 hours while the optimal solution is $Q = 137$ with a cycle time of 45.6 hours, well within the error of our parameter estimates. However, the near optimal solution is 27.9 hours (over two days, at 13.7 hours per day) less than the EOQ solution.

5 Discussion

So which method is better, EOQ which considers total "cost" or the cycle time method? Note that EOQ focuses only on inventory carrying cost (using a crude estimate of inventory, $Q/2$) and setup "cost." One problem with setup cost is that if we reduce the number of setups do we really reduce total cost? If we don't fire anyone or increase throughput (set by demand), there is no change in cost or revenue. So what is setup cost? In reality, we know that we cannot be setting up all the time and so we like to assign a ficticious cost to prevent this behavior. If the plant can do the extra setups required by the minimum cycle time solution without hiring additional people, it will obviously be the right choice. However, if there really is an increase in cost, we may need to reconsider.

Compare the total "cost" of the cycle time solution with the EOQ solution. Assuming there really is a setup cost then the total unit cost will be \$150 + \$250/416 = \$150.60. For the minimum cycle time solution, the cost will be $150 + 250/160 =$ \$151.58 or \$0.98 (or 1%) more than the EOQ solution. If this cost is real (i.e., if we really do have to hire more people to do setups more frequently) then we must consider whether the extra dollar per unit is worth the two days saved in cycle time. Recalling that reducing cycle time causes also tends to improve quality, reduce forecasting, reduce WIP and finished goods inventory, and improve the flexibility of the system, it might be well worth the extra expense.

If we were to reduce setups from 10 hours to 2 hours, the setup cost would reduce to \$50 per setup and the EOQ would become 186 with a setup cost per part of \$0.27 and a cycle time of 27.9 hours (about 2 days). The minimum CT solution would yield a lot size of 32 parts with a setup cost per part of \$1.56 (not much different than the previous case) and a cycle time of 9.31 hours (around 0.7 days).

Factory Physics principles state that variability always degrades performance and that there will be a buffer of some sort to accommodate. There are only three kinds of buffers: capacity, time, and inventory. This is illustrated here very clearly. Because of the need to have a setup, there is variability in process time (e.g., the first part takes 10 hours and 6 minutes, while the next several hundred parts take only 6 minutes). The EOQ uses a large time buffer (and, correspondingly a large WIP buffer) to accommodate this variability while the minimum CT solution uses a large capacity buffer (i.e., more capacity is used up in setups) and a smaller time and WIP buffer. Trying to decide what is best depends on how important is short cycle times and low WIP as compared to low labor cost.

Many times, however, the true savings of short cycle times and less WIP levels that result from less rework, less scrap, and better forecasting. If we can detect bad parts before many have been made, there can be an enormous savings. Likewise, if the cycle times are shorter, forecasting will be improved or, possibly, even eliminated. Many times these savings can be much greater than the labor "savings" or larger lot sizes.

In conclusion, determining best lot sizes can be a tricky proposition. However, it appears that the cost of long cycle times is understated in most accounting systems. Thus, using lot sizes that minimize cycle times may be a good alternative to the traditional Economic Order Quantity.